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AN EFFECTIVE METHOD OF SOLVING DYNAMIC PROBLEMS FOR LAYERED MEDIA WITH DISCONTINUOUS BOUNDARY CONDITIONS†

O. D. PRYAKHINA and A. V. SMIRNOVA

Krasnodar

e-mail: donna@kubsu.ru

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A method of determining the dynamic characteristics of multilayered semi-bounded media, using a special representation of the solution for one layer and applicable for an arbitrary number of layers [1], is extended to the case of media with defects of the crack-cavity type at the interfaces of the layers. Functional-matrix relations are obtained which enable a system of integral equations, connecting the jumps in the displacements and stresses on the sides of the cracks, to be written. Examples of the use of these relations for certain special types of media are given. © 2004 Elsevier Ltd. All rights reserved.

When investigating the dynamic modes of oscillation of layered semi-bounded media, the form of the functional-matrix relations, which are the basis of constructing integral equations and their systems, which connect the displacements and stresses, is of considerable importance. The main difficulties in the numerical realization of existing methods of solving problems for multilayered media are due to the presence of increasing exponential components in the fundamental solutions of the corresponding systems of differential equations, which lead to instability of the numerical procedures for solving the boundary-value problem and make the linear algebraic systems, which arise when satisfying the boundary conditions, ill-posed. All these approaches require solutions of high-order systems of equations, and the greater the number of layers the greater the difficulties of a computational nature which arise. To overcome these, a number of methods have been developed, a review and a comparative analysis of which are given in [1–4]. For example, a method was proposed in [2, 5], the stability of which is ensured by separating the exponential components and taking them outside the frame of the computational process. Solutions have been constructed for multilayered media in problems of statics [6].

However, it cannot be assumed that all the problems are thereby removed and optimal algorithms have been developed covering all changes in the parameters of the problem.

The use of these approaches is complicated considerably or becomes impossible when describing the dynamic behaviour of elastic semibounded inhomogeneous media, containing defects like cracks or inclusions. Moreover, the pressing need to investigate problems in this formulation is dictated by their practical importance in seismology, seismic prospecting, flaw detection, electronics and other areas.

New formulations of this kind of problem have been presented, a classification of the inhomogeneities has been given, and a method of solution has been proposed, and also, the conditions for the localization of the vibration process of a system of cracks and/or inclusions were proposed for first time in [7, 8].

Below we propose an effective analytical method of constructing a solution of dynamic problems for layered media when discontinuous conditions are imposed on the layer interfaces. The method is based on the use of a special representation of the solution for one layer, and it is applicable for an arbitrary number of layers and arrangements of the inhomogeneities. The advantage of this method is the possibility of constructing simple algorithms of numerical analysis, applicable for a wide range of variation of the parameters of the problem. Unlike existing approaches, it does not require a numerical solution of high-order algebraic systems, which arise when satisfying the boundary conditions, and it does not contain increasing exponential components in the representation of the solution obtained. A solution of the problem for a uniform semibounded medium (a layer or half-space), containing a system of plane parallel-oriented crack-cavities is then obtained as a special case, if we assume the physical-mechanical parameters of the layers to be alike.

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1. OSCILLATIONS OF A PACKET OF LAYERS

Suppose the medium consists of a packet of N plane-parallel layers of thickness $H = 2(h_1 + h_2 + \ldots + h_N)$ with a rigidly clamped lower face and occupies a region $-H \leqslant z \leqslant 0$, $-\infty \leqslant x, y \leqslant +\infty$ (h_k is the half-thickness of the kth layer). The surface of the medium is subjected to a certain dynamic force, characterized by the vector $\mathbf{t}_0(x, y, t)$, having as its components the shear stresses t_{10} and t_{20} and normal stresses t_{30}.

We will introduce a local system of coordinates for each layer

$$z_k = z + 2\sum_{i=1}^{k-1} h_i + h_k, \quad k = 1, 2, \ldots, N$$

Then the solution for the kth layer in terms of Fourier transforms (for the harmonic problem) or Fourier–Laplace transforms (for the non-stationary problem) will be defined by the expression [3, 9]

$$\mathbf{W}_k(z_k) = \frac{1}{\mu_k}[\mathbf{B}_+(z_k)\mathbf{T}_{k-1} + \mathbf{B}_-(z_k)\mathbf{T}_k], \quad -h_k \le z_k \le h_k \tag{1.1}$$

where

$$\mathbf{T}_0 = LF\mathbf{t}_0, \quad \mathbf{T}_k = LF\mathbf{t}_k, \quad \mathbf{W}_k = LF\mathbf{w}_k$$

μ_k is the shear modulus of the kth layer, F is the two-dimensional Fourier transformation operator with respect to the variables x, y, L is the Laplace operator with respect to time t, $\mathbf{t}_k = \{t_{1k}, t_{2k}, t_{3k}\}$ are the stress vectors, characterizing the interaction between the layers, and $\mathbf{w}_k = \{w_{1k}, w_{2k}, w_{3k}\}$ is a vector whose components are the horizontal displacements w_{1k} and w_{2k} and vertical displacement w_{3k} of points of the kth layer.

We will assume that there are discontinuous boundary conditions for the displacements along the interfaces of the layers. We will write the condition on the lower face of the packet of layers

$$\mathbf{W}_N(-h_N) = 0 \tag{1.2}$$

and the conditions for the layers to adjoin

$$\mathbf{W}_k(-h_k) = \mathbf{W}_{k+1}(h_{k+1}) + \mathbf{f}_k, \quad k = 1, 2, \ldots, N-1 \tag{1.3}$$

where $\mathbf{f}_k$ $\{f_{1k}, f_{2k}, f_{3k}\}$ is the vector of the jump in the displacements at the interfaces of the layers.

Condition (1.3), taking expression (1.1) into account, leads to a recurrence relation connecting the characteristics of the stress–strain state the layers with the parameters of their contact interaction

$$\mathbf{B}_+(-h_k)\mathbf{T}_{k-1} + \mathbf{B}_-(-h_k)\mathbf{T}_k = g_k[\mathbf{B}_+(h_{k+1})\mathbf{T}_k + \mathbf{B}_-(h_{k+1})\mathbf{T}_{k+1}] + \mu_k\mathbf{f}_k \tag{1.4}$$

We will introduce the vectors Λ_k and the matrices $\mathbf{F}_k$

$$\mathbf{\Lambda}_k = \mathbf{D}_{k+1}\mathbf{\Lambda}_{k+1} + \mu_k\mathbf{f}_k, \quad \mathbf{F}_k = \mathbf{B}_-(-h_k) - g_k\mathbf{B}_+(h_{k+1}) - \mathbf{D}_{k+1}\mathbf{G}_{k+1} \tag{1.5}$$

$$k = 1, 2, \ldots, N-1$$

$$\mathbf{\Lambda}_N = 0, \quad \mathbf{F}_N = \mathbf{B}_-(-h_N)$$

Here

$$\mathbf{G}_k = -\mathbf{B}_+(-h_k), \quad \mathbf{D}_{k+1} = g_k\mathbf{B}_-(h_{k+1})\mathbf{F}_{k+1}^{-1}$$

From condition (1.2) we obtain

$$\mathbf{F}_N\mathbf{T}_N = \mathbf{G}_N\mathbf{T}_{N-1} \tag{1.6}$$

Then the recurrence relations (1.4) for determining the stress vectors between the layers, taking equality (1.6) and notation (1.5) into account, can be written in the form

$$\mathbf{F}_k\mathbf{T}_k = \mathbf{G}_k\mathbf{T}_{k-1} + \mathbf{\Lambda}_k, \quad k = 1, 2, \ldots, N$$

Using the last relations, assuming $k = 1, 2, \ldots, N$ in succession, we determine the forces acting on the kth layer $\mathbf{T}_k$ due to the surface load $\mathbf{T}_0$ and the jumps in the displacements $\mathbf{f}_k$

$$\mathbf{T}_k = \prod_{i=k}^{1}(\mathbf{F}_i^{-1}\mathbf{G}_i)\mathbf{T}_0 + \sum_{m=1}^{k}\prod_{i=k}^{m}(\mathbf{F}_i^{-1}\mathbf{G}_i)\mathbf{G}_m^{-1}\mathbf{\Lambda}_m, \quad k = 1, 2, \ldots, N \tag{1.7}$$

The matrices Λ_k are given by the formulae (1.5).

Substituting the expressions obtained for the forces (1.7) into (1.1), we determined the displacements of the points of the medium in the kth layer

$$\mathbf{W}_k(z_k) = \frac{1}{\mu_k}((\mathbf{B}_+(z_k) + \mathbf{B}_-(z_k)\mathbf{F}_k^{-1}\mathbf{G}_k)\mathbf{\Pi}_k + \mathbf{B}_-(z_k)\mathbf{F}_k^{-1}\mathbf{\Lambda}_k), \quad k = 1, 2, \ldots, N \tag{1.8}$$

where

$$\mathbf{\Pi}_k = \prod_{i=k-1}^{1}(\mathbf{F}_i^{-1}\mathbf{G}_i)\mathbf{T}_0 + \sum_{m=1}^{k-1}\prod_{i=k-1}^{m}(\mathbf{F}_i^{-1}\mathbf{G}_i)\mathbf{G}_m^{-1}\mathbf{\Lambda}_m$$

If the surface is stress-free, we have

$$\mathbf{T}_k = \sum_{m=1}^{k}\left(\prod_{i=k}^{m}\mathbf{F}_i^{-1}\mathbf{G}_i\right)\mathbf{G}_m^{-1}\mathbf{\Lambda}_m$$

$$\mathbf{W}_k(z_k) = \frac{1}{\mu_k}\left((\mathbf{B}_+(z_k) + \mathbf{B}_-(z_k)\mathbf{F}_k^{-1}\mathbf{G}_k)\sum_{m=1}^{k-1}\prod_{i=k-1}^{m}(\mathbf{F}_i^{-1}\mathbf{G}_i)\mathbf{G}_m^{-1}\mathbf{\Lambda}_m + \mathbf{B}_-(z_k)\mathbf{F}_k^{-1}\mathbf{\Lambda}_k\right)$$

Relations (1.7) enables us to construct a system of integral equations connecting the displacements and stresses of the sides of the cracks, situated at the joint of the layers in sections of these cracks.

Assuming $\mathbf{f}_k = 0$ in relations (1.7) and (1.8), we arrive at the case of ideal contact between the layers

$$\mathbf{T}_k = \prod_{i=k}^{1}(\mathbf{F}_i^{-1}\mathbf{G}_i)\mathbf{T}_0, \quad \mathbf{W}_k(z_k) = \frac{1}{\mu_k}\mathbf{K}(z_k)T_0$$

The matrix function

$$\mathbf{K}(z_k) = (\mathbf{B}_+(z_k) + \mathbf{B}_-(z_k)\mathbf{F}_k^{-1}\mathbf{G}_k)\prod_{i=k-1}^{1}(\mathbf{F}_i^{-1}\mathbf{G}_i)$$

is Green's matrix symbol, corresponding to the dynamic problem.

The matrix functions $\mathbf{B}_\pm(z)$ for different types of media (elastic, electroelastic, anisotropic, thermoelastic and thermoelectroelastic) have a different structure and were given previously in [3]. For example, for an anisotropic layer in the case of the harmonic problem

$$\mathbf{B}_\pm(z) = \begin{Vmatrix} \alpha^2 m_1^\pm + \beta^2 n^\pm & \alpha\beta(m_1^\pm - n^\pm) & \pm i\alpha m_2^\pm \\ \alpha\beta(m_1^\pm - n^\pm) & \beta^2 m_1^\pm + \alpha^2 n^\pm & \pm i\beta m_2^\pm \\ -i\alpha k_1^\pm & -i\beta k_1^\pm & \pm k_2^\pm \end{Vmatrix} \tag{1.9}$$

$$m_i^\pm = M_i^- \pm M_i^+, \quad n^\pm = N^- \pm N^+, \quad k_i^\pm = K_i^- \pm K_i^+; \quad i = 1, 2$$

The elements K_i^-, M_i^-, N^-, Δ^- correspond to the skew-symmetric problem for a single layer and are obtained from the corresponding elements K_i^+, M_i^+, N^+, Δ^+ of the symmetric problem by making the replacement sh $\leftrightarrow$ ch.

In the special case of an isotropic medium

$$M_1^+ = \sigma_2[\gamma c_2(z) - \lambda^2 c_1(z)]/(\lambda^2\Delta^+), \quad M_2^+ = [\sigma_1\sigma_2 t_1 c_2(z) - \gamma t_2 c_1(z)]/\Delta^+$$

$$K_1^+ = [\sigma_1\sigma_2 s_1(z) - \gamma s_2(z)]/\Delta^+, \quad N^+ = \operatorname{ch}\sigma_2 z/(2\lambda^2\sigma_2\operatorname{sh}\sigma_2 h)$$

$$\begin{gathered} K_2^+ = \sigma_1[\gamma s_1(z)t_2 - \lambda^2 t_1 s_2(z)]/\Delta^+, \quad \Delta^+ = 4(\gamma^2 t_2 - \lambda^2\sigma_1\sigma_2 t_1) \\ s_i(z) = \operatorname{sh}\sigma_i z/\operatorname{ch}\sigma_i h, \quad c_i(z) = \operatorname{ch}\sigma_i z/\operatorname{ch}\sigma_i h, \quad t_i = \operatorname{th}\sigma_i h; \quad i = 1, 2 \end{gathered} \tag{1.10}$$

$$\gamma = \lambda^2 - \Omega^2/2, \quad \Omega^2 = \rho\omega^2 a^2/\mu, \quad \lambda^2 = \alpha^2 + \beta^2$$

$$\sigma_2^2 = \lambda^2 - \Omega^2, \quad \sigma_1^2 = \lambda^2 - \varepsilon\Omega^2, \quad \varepsilon = (1-2\nu)/(2-2\nu)$$

where σ_j are the roots of the characteristic equation of the problem for a single layer, Ω is the reduced frequency of the oscillations, a is a characteristic linear dimension (for example, the areas of action of the surface load), ρ is the density, μ is the shear modulus, ν is Poisson's ratio for a specific layer, occupying the region ($|z| \leqslant h, -\infty < x, y, < \infty$), and α and β are parameters of the Fourier transformation.

Note that when calculating the elements of the matrices $\mathbf{B}_\pm(z_k)$ in formulae (1.10) it is necessary to take the corresponding parameters of the kth layer (μ_k, ρ_k, ν_k, h_k).

We will give examples of the use of the recurrence relations for certain special cases.

1. When $N = 1$ it follows from expressions (1.7) and (1.8) that

$$\mathbf{T}_1 = \mathbf{F}_1^{-1}\mathbf{G}_1\mathbf{T}_0, \quad \mathbf{W}_1(z_1) = \frac{1}{\mu_1}(\mathbf{B}_+(z_1) + \mathbf{B}_-(z_1)\mathbf{F}_1^{-1}\mathbf{G}_1)\mathbf{T}_0, \quad z_1 = z + h_1$$

$$\mathbf{F}_1 = \mathbf{B}_-(-h_1), \quad \mathbf{\Lambda}_1 = 0$$

which corresponds to the problem of the oscillations of a layer rigidly attached to a non-deformable base.

2. When $N = 2$ we obtain from (1.7) the following expressions for the stresses

$$\mathbf{T}_1 = \mathbf{F}_1^{-1}(\mathbf{G}_1\mathbf{T}_0 + \mathbf{\Lambda}_1), \quad \mathbf{T}_2 = \mathbf{F}_2^{-1}\mathbf{G}_2\mathbf{F}_1^{-1}(\mathbf{G}_1\mathbf{T}_0 + \mathbf{\Lambda}_1) \tag{1.11}$$

and from Eq. (1.8) we obtain the following expression for the displacements in the upper layer

$$\mathbf{W}_1(z_1) = \frac{1}{\mu_1}((\mathbf{B}_+(z_1) + \mathbf{B}_-(z_1)\mathbf{F}_1^{-1}\mathbf{G}_1)\mathbf{T}_0 + \mathbf{B}_-(z_1)\mathbf{F}_1^{-1}\mathbf{\Lambda}_1), \quad z_1 = z + h_1 \tag{1.12}$$

and in the lower layer

$$\mathbf{W}_2(z_2) = \frac{1}{\mu_2}(\mathbf{B}_+(z_2) + \mathbf{B}_-(z_2)\mathbf{F}_2^{-1}\mathbf{G}_2)\mathbf{F}_1^{-1}(\mathbf{G}_1\mathbf{T}_0 + \mathbf{\Lambda}_1), \quad z_2 = z + 2h_1 + h_2 \tag{1.13}$$

In this case

$$\mathbf{F}_1 = \mathbf{B}_-(-h_1) - g_1\mathbf{B}_+(h_2) - g_1\mathbf{B}_-(h_2)\mathbf{F}_2^{-1}\mathbf{G}_2, \quad \mathbf{F}_2 = \mathbf{B}_-(-h_2), \quad \mathbf{\Lambda}_1 = \mu_1\mathbf{f}_1$$

3. For $N = 3$ we will obtain formulae for the displacements for a three-layer base. The displacements of points of the upper layer are described by formula (1.12), the displacements of points of the second layer are described by formula (1.13) with the addition of the term $\mathbf{B}_-(z_2)\mathbf{F}_2^{-1}\mathbf{\Lambda}_2$ to its right-hand side, while the displacements of the points of the third layer are described by the relation

$$\mathbf{W}_3(z_3) = \frac{1}{\mu_3}((\mathbf{B}_+(z_3) + \mathbf{B}_-(z_3)\mathbf{F}_3^{-1}\mathbf{G}_3)\mathbf{F}_2^{-1}\mathbf{G}_2\mathbf{F}_1^{-1}(\mathbf{G}_1\mathbf{T}_0 + \mathbf{\Lambda}_1) +$$

$$+ (\mathbf{B}_+(z_3) + \mathbf{B}_-(z_3)\mathbf{F}_3^{-1}\mathbf{G}_3)\mathbf{F}_2^{-1}\mathbf{\Lambda}_2), \quad z_3 = z + 2h_1 + 2h_2 + h_3$$

The general form of the matrix $\mathbf{F}_1$ in this case is identical with the form of the analogous matrix for case 2, while

$$\mathbf{F}_2 = \mathbf{B}_-(-h_2) - g_2\mathbf{B}_+(h_3) - g_2\mathbf{B}_-(h_3)\mathbf{F}_3^{-1}\mathbf{G}_3, \quad \mathbf{F}_3 = \mathbf{B}_-(-h_3)$$

$$\mathbf{\Lambda}_1 = \mu_1\mathbf{f}_1 + \mathbf{D}_2\mu_2\mathbf{f}_2, \quad \mathbf{\Lambda}_2 = \mu_2\mathbf{f}_2$$

2. THE OSCILLATION OF A LAYERED HALF-SPACE

The solution of the problem of a multilayered medium, rigidly attached to an elastic half-space, is easily obtained by letting the thickness of the lower layer ($k = N$) tend to infinity, and replacing the system of coordinates as follow $z^* = z_N - h_N$. Taking the limit we obtain

$$\mathbf{F}_{N-1} = \mathbf{B}_-(-h_{N-1}) - g_{N-1}\mathbf{B}_+^\infty(0)$$

$$\mathbf{F}_k = \mathbf{B}_-(-h_k) - g_k\mathbf{B}_+(h_{k+1}) - \mathbf{D}_{k+1}\mathbf{G}_{k+1}, \quad k = 1, 2, \ldots, N-2$$

We obtain the stresses acting on the joint of the layers in the form (1.7), where $k = 1, 2, \ldots, N-1$, while $\mathbf{T}_N = 0$. To calculate the displacements of an arbitrary point of the medium we have

$$\mathbf{W}_k(z_k) = \frac{1}{\mu_k}((\mathbf{B}_+(z_k) + \mathbf{B}_-(z_k)\mathbf{F}_k^{-1}\mathbf{G}_k)\mathbf{\Pi}_k + \mathbf{B}_-(z_k)\mathbf{F}_k^{-1}\mathbf{\Lambda}_k) \tag{2.1}$$

$$z_k = z + 2\sum_{i=1}^{k-1} h_i + h_k, \quad k = 1, 2, \ldots, N-1$$

$$\mathbf{W}_N(z^*) = \frac{1}{\mu_N}\mathbf{B}_+^\infty(z^*)\mathbf{\Pi}_N, \quad z^* = z + 2\sum_{i=1}^{N-1} h_i$$

The form of Π_k was derived above.

Since when $h_N \to \infty$ we have

$$m_i^- = n^- = k_i^- = 0, \quad m_i^+ = 2M_i^+ = m_i^0, \quad n^+ = 2N^+ = n^0, \quad k_i^+ = 2K_i^+ = k_i^0$$

then

$$\mathbf{B}_-^\infty(z^*) \equiv 0, \quad \mathbf{B}_+^\infty(z^*) = \lim_{h_N\to\infty}\mathbf{B}_+(z_N) = \lim_{h_N\to\infty}\mathbf{B}_+(z^* + h_N) =$$

$$= \left\| \begin{matrix} \alpha^2 m_1^0 + \beta^2 n^0 & \alpha\beta(m_1^0 - n^0) & \pm i\alpha m_2^0 \\ \alpha\beta(m_1^0 - n^0) & \beta^2 m_1^0 + \alpha^2 n^0 & \pm i\beta m_2^0 \\ -i\alpha k_1^0 & -i\beta k_1^0 & \pm k_2^0 \end{matrix} \right\|$$

If the underlying half-space is isotropic, the quantities occurring in the matrix $\mathbf{B}_+^\infty(z)$ have the form

$$m_1^0 = 2\sigma_2(-\lambda^2 e^{\sigma_1 z} + \gamma e^{\sigma_2 z})/(\lambda^2\Delta), \quad n^0 = e^{\sigma_2 z}/(\lambda^2\sigma_2)$$

$$m_2^0 = 2(-\gamma e^{\sigma_1 z} + \sigma_1\sigma_2 e^{\sigma_2 z})/\Delta$$

$$k_1^0 = 2(-\gamma e^{\sigma_2 z} + \sigma_1\sigma_2 e^{\sigma_1 z})/\Delta, \quad k_2^0 = 2\sigma_1(-\lambda^2 e^{\sigma_2 z} + \gamma e^{\sigma_1 z})/\Delta$$

$$\Delta = 4(\gamma^2 - \lambda^2\sigma_1\sigma_2), \quad \sigma_i^2 = \lambda^2 - \Omega_i^2, \quad i = 1, 2, \quad \gamma = \lambda^2 - \Omega_2^2/2$$

$$\Omega_1^2 = \Omega_2^2(1 - 2\nu_N)/(2 - 2\nu_N), \quad \Omega_2^2 = \rho_N\omega^2 a^2/\mu_N$$

(ρ_N, μ_N and ν_N are the density, shear modulus and Poisson's ratio of the half-space).

It can be seen from the above formulae that the condition $\mathbf{W}(z) \to 0, z \to -\infty$ is satisfied in the case of a layered half-space.

In particular, for a uniform half-space we obtain the simple formula

$$\mathbf{W}(z) = \frac{1}{\mu_1}\mathbf{B}_+^\infty(z)\mathbf{T}_0 \quad (z \le 0)$$

For a layer rigidly attached to a half-space, the stresses $\mathbf{T}_1$ on the layer interface and the half-space have the form (1.11), while the displacements in the layer ($-2h_1 \leqslant z \leqslant 0$) have the form (1.12), but in this case

$$\mathbf{F}_1 = \mathbf{B}_-(-h_1) - g_1\mathbf{B}_+^\infty(0)$$

The displacements in the half-space ($z \leqslant -2h_1$) will be described by the expression

$$\mathbf{W}_2(z) = \frac{1}{\mu_2}\mathbf{B}_+^\infty(z+2h_1)\mathbf{F}_1^{-1}(-\mathbf{B}_+(-h_1)\mathbf{T}_0 + \mu_1\mathbf{f}_1)$$

Applying an inverse Fourier transformation to relation (1.8), we obtain an integral representation of the solution for the harmonic problem (the factor $e^{-i\omega t}$ is omitted)

$$\mathbf{w}(x, y, z, \omega) \equiv \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{W}(z)e^{-i(\alpha x + \beta y)}d\alpha d\beta \tag{2.2}$$

$$\mathbf{W}(z) \equiv \mathbf{W}(\alpha, \beta, z_k, \omega), \quad z = z_k - 2\sum_{i=1}^{k-1} h_i - h_k, \quad k = 1, 2, \dots, N$$

For the non-stationary problem it is necessary to put $\omega = ip$ in Eq. (2.2) (p is the parameter of the Laplace transformation) and to apply an inverse Laplace transformation. It was shown in [3, 10] that the solution of the non-stationary problem can also be represented in the form

$$\mathbf{w}(x, y, z, t) = \frac{2}{\pi}\int_0^\infty \mathrm{Re}[\mathbf{w}(x, y, z, \omega)]\cos(\omega t)d\omega$$

Remark. We will assume that it is necessary to determine the displacements of points of the medium at a depth $z = z_0$. Then, if

$$2\sum_{i=1}^{s-1} h_i < |z_0| \le 2\sum_{i=1}^{s} h_i$$

for numerical calculations in expressions (1.8) or (2.1) one must assume $k = s$ ($s \leqslant N$).

3. OSCILLATIONS OF A LAYER OR OF A HALF-SPACE WITH DEFECTS

The solution of the problem for a homogeneous semibounded medium (a layer or half-space), containing $N - 1$ plane parallel-oriented cavities or cracks, is described by the functional-matrix relations for the stresses (1.7) and the displacements (1.8) and (2.1), in which we must assume the physical-mechanical parameters to be alike for all k ($k = 1, 2, \dots, N$).

Thus, assuming $\mu_{1,2} = \mu$, $\rho_{1,2} = \rho$, $\nu_{1,2} = \nu$, $h_{1,2} = h$, we obtain from relations (1.7) for $\mathbf{T}_0 = 0$ and $N = 2$ a solution of the harmonic three-dimensional problem for a single crack, situated in a homogeneous layer of thickness $H = 4h$ at the same distance $2h$ from its boundaries,

$$\mathbf{T}_1 = \mu\mathbf{F}_1^{-1}\mathbf{f}_1 \tag{3.1}$$

where

$$\mathbf{F}_1 = \mathbf{B}_-(-h) - \mathbf{B}_+(h) + \mathbf{B}_-(h)\mathbf{B}_-^{-1}(-h)\mathbf{B}_+(-h)$$

$$\mathbf{f}_1(\alpha, \beta, \Omega) = \Delta\mathbf{W}(\alpha, \beta, \Omega)$$

$$\Delta\mathbf{W} = \mathbf{W}^+ - \mathbf{W}^-, \quad \mathbf{W}^+ = \mathbf{W}_1(-h_1) = \mathbf{W}(-h), \quad \mathbf{W}^- = \mathbf{W}_2(h_2) = \mathbf{W}(h)$$

Here it is assumed that the sides of the cracks do not interact and that the stresses on these sides are equal to $\mathbf{T}_1 = \mathbf{T}^+ = \mathbf{T}^-$ (i.e. there are static stresses which "open up" the sides of the crack).

The functional matrix relation (3.1) enables us to write a system of integral equations for the jump in the displacements $\Delta\mathbf{w}(x, y)$ on the sides of the crack

$$\iint\limits_S \mathbf{k}(x-\xi, y-\eta)\Delta\mathbf{w}(\xi, \eta)d\xi d\eta = \frac{\mathbf{t}_1}{\mu} \quad (x, y) \in S$$

$$\mathbf{k}(x, y) = \frac{1}{4\pi^2}\int\limits_{\delta_1}\int\limits_{\delta_2} \mathbf{F}_1^{-1}(\alpha, \beta, \Omega)e^{-i(\alpha x + \beta y)}d\alpha d\beta$$

where δ_1 and δ_2 are the contours of integration, the rules for choosing which were given in [11], and S is the region occupied by the crack.

The further solution of this system requires the use of the method of fictitious absorption, the factorisation method or numerical methods [2, 3, 11].

Hence, the proposed approach enables one to model any combination of continuous and discontinuous conditions at the interfaces of the layers. Moreover, the advantage of the use of this representation for each layer is that it is possible to investigate media with an arbitrary number of layers, each of which can possess complex physical-mechanical properties.

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